

First Observations of Nonhydrodynamic Mix at the Fuel-Shell Interface in Shock-Driven Inertial Confinement Implosions

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A strong nonhydrodynamic mechanism generating atomic fuel-shell mix has been observed in strongly shocked inertial confinement fusion implosions of thin deuterated-plastic shells filled with ^3He gas. These implosions were found to produce D^3He -proton shock yields comparable to implosions of identical shells filled with a hydroequivalent 50:50 D^3He gas mixture. Standard hydrodynamic mixing cannot explain this observation, as hydrodynamic modeling including mix predicts a yield an order of magnitude lower than was observed. Instead, these results can be attributed to ion diffusive mix at the fuel-shell interface.

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The impact of hydrodynamic mix on inertial confinement fusion (ICF) implosions has been a subject of study since their inception [1–8]. Mix of nonhydrogenic materials into the fuel is severely detrimental to the final pressure of the fuel assembly, by enhancing the energy loss pathways of bremsstrahlung and line radiation [2]. The study of mix has previously concentrated on the growth of hydrodynamic instabilities in accelerating frames, such as Rayleigh-Taylor, Kelvin-Helmholtz, and Richtmyer-Meshkov instability growth at the ablation front [3,4] and during deceleration [5,6]. Instabilities at interfaces have been studied as a seed for deceleration phase growth [7,8]. Because of low convergence and rapid total ablation (or “burnthrough”) of the shell, shock driven “exploding pusher” implosions are expected to have minimal growth of hydrodynamic instabilities prior to peak neutron production [9]. The experiments presented in this Letter, however, demonstrate significant fuel-shell mix prior to the deceleration phase, an epoch during which hydrodynamic instability growth is expected to be negligible. To explain these results, we propose that ion diffusion, a kinetic process, is generating substantial mix at the fuel-shell interface. To the best of our knowledge, ion diffusion has not previously been recognized or studied as a significant contributor to atomic mix in ICF.

The experiments described in this Letter were performed at the 60-beam OMEGA laser system [10]. Spherical capsules, 860 μm in diameter and made of 5- μm -thick deuterated plastic (CD) [11], were filled with mixtures of deuterium and ^3He gas and imploded with a 1 ns square laser pulse delivering 30 kJ of laser energy, which launches a strong shock into the fuel. Such thin-shell exploding pusher capsules are expected to produce nuclear yield

primarily from heating of the fuel by the shock after it converges and rebounds (shock yield), rather than from compression of the fuel by the shell material (compression yield), as the remaining mass of the CD shell after shock convergence is too small to significantly compress and heat the fuel. Gas fills of pure deuterium, 50:50 D^3He , and pure ^3He were used, while maintaining a constant mass density of 0.49 mg/cc. Maintaining constant mass density produces plasmas with equivalent equations of state for any D^3He ratio [12], such that the experiments will evolve similarly when subjected to the same pressure source from the laser [13].

Earlier experiments imploded much thicker 20- μm plastic shells with an inner layer of 1 μm CD [14,15]. When filled with a 50:50 mixture of D and ^3He fuel, these targets produced a measurable D^3He -proton yield at both shock and compression bang time. However, hydroequivalent pure ^3He -filled targets produced no observable yield at shock burn. The lack of mix at shock burn is expected, since the fuel-shell interface does not become unstable to Rayleigh-Taylor growth until deceleration of the remaining shell mass is initiated by the rebounding shock striking the interface. Similar work has been performed in compressively driven thick CD targets filled with T_2 fuel [16,17].

Based in part on these previous findings, the expectation for the shock-driven experiments presented in this Letter was that fuel-shell mix at shock bang time would be minimal and thus the shock yield of D^3He protons would be small for the pure ^3He -filled targets. However, the opposite was observed, as is shown in Fig. 1. The yield of D^3He protons from pure ^3He -filled targets was essentially identical to the yield from targets filled with a 50:50 D^3He mixture and imploded with identical laser

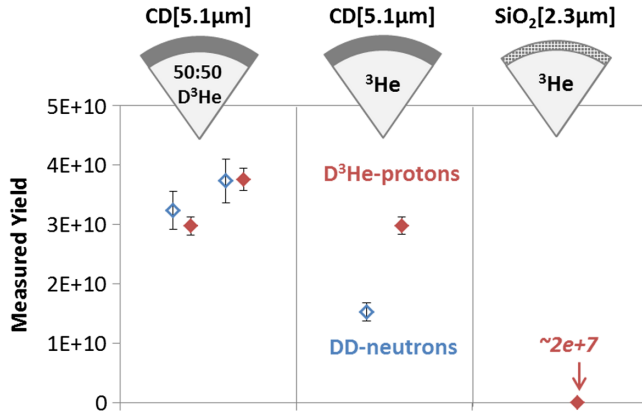


FIG. 1 (color online). Yields of D^3He protons (solid red) recorded from 30 kJ implosions of deuterated plastic shells filled with a 50:50 D^3He mixture are comparable in magnitude to implosions of the same shells filled with pure 3He . Implosions of glass shells filled with 3He produce yields 3 orders of magnitude lower, ruling out D_2 -gas contamination of the targets as an explanation for this result. The observed D^3He - p yields require shell-deuterium mix into the 3He fuel to be of the order of 10% the initial 3He gas density.

conditions. Follow-up 3He -filled implosions performed with reduced laser energy (23 kJ) and full beam smoothing generated the same results, producing DD- n and D^3He - p yields comparable to the 30 kJ implosions.

The nuclear yields of 14.7 MeV protons from the D - 3He fusion reaction were measured using multiple wedge range-filter proton spectrometers and the charged particle spectrometers (CPS1 and CPS2) [18]. D^3He -proton yields above 10^{10} were produced by capsules containing 50:50 D - 3He mixtures and containing pure 3He . Yields of 2.45 MeV neutrons from the D - D fusion reaction were measured above 10^{10} , using the neutron time-of-flight (nTOF) diagnostic suite [19]. A nuclear bang time of 780 ± 50 ps was recorded on a hydroequivalent D_2 -filled implosion using the neutron temporal diagnostic [20].

The observed D^3He -proton and DD-neutron yields were used to estimate how much deuterium would have to enter the pure 3He fuel prior to burn to produce these results. The D^3He -proton yield scales as

$$Y_{D^3He-p} = \int n_D n_{^3He} \langle \sigma v \rangle_{D^3He} dV dt. \quad (1)$$

The evolution of density and temperature profiles is to zeroth order identical in both the 50:50 D : 3He and pure 3He case, so the yield is proportional primarily to the number density of the reactant ions. Assuming a uniform distribution of mix, the mixed deuterium number density in the pure 3He fuel (n'_D) can be estimated as

$$n'_D = \left(\frac{n_{^3He,0.5}}{n_{^3He,1}} \right) n_{D,0.5} = 0.36 n_{^3He,1}, \quad (2)$$

where the subscript number indicates the initial atomic 3He ion fraction in the gas. This value corresponds to the inner 47 nm of initial CD material being uniformly mixed throughout the 3He fuel. As this density is sufficiently high to perturb the hydrodynamic evolution of the plasma, the calculation is approximate; however, it is instructive to note the large amount of deuterium mix that is required to explain the observations. The DD-neutron yield produced by this level of mixed deuterium follows from similar scalings as 0.36 times the DD-neutron yield from the fuel in the 50:50 experiment, consistent with observations. However, in both experiments, the neutron yield produced in the CD plasma when it is reshocked is expected to produce of order 10^{10} DD neutrons [21].

Mixing of the shell and fuel mass is the best candidate for explaining these observations. Contamination of the 3He gas with deuterium has been ruled out as a cause for this observation, as the gas source of 3He was determined to contain 1.4 parts atomic D per 10^4 atomic 3He by mass spectroscopy. A 3He -filled glass target with similar initial shell ρR was imploded as a control experiment, and produced D^3He -proton shock yield 3 orders of magnitude lower than was observed from the CD-shell experiments, as shown on the right of Fig. 1.

Permeation of 3He gas into the CD shell prior to the shot is ruled out as a cause for this observation. Assuming the shell maintains a 3He gas partial-pressure equal to the gas fill pressure, the D : 3He ratio in the shell is 425:1. Including this value in 1D-radiation-hydrodynamic simulations produces yields approximately 2 orders of magnitude below those observed.

While hydrodynamic instabilities are expected to be a negligible contributor to mix in experiments dominated by shock yield, given the long history of discussion behind hydrodynamic sources of mix it is important to preclude these mechanisms. To investigate the growth of hydrodynamic modes in detail, multimode 2D-DRACO simulations were performed [22], as shown in Fig. 2. In these simulations, which explored modes up to $\ell = 150$ and included the effect of laser imprint, instability growth at the fuel-shell interface was negligible through shock rebound [23]. The large ablation velocity (approximately $10 \mu\text{m/ns}$) likely stabilizes the growth of ablation-front instability. After the shell burns through at 0.5 ns, no steep gradients remain where the Atwood number is large and the ablation-front instability growth is truncated.

The shock breakout through the fuel-shell interface does not drive significant amounts of mix due to the Richtmyer-Meshkov (RM) instability. The RM instability grows linearly with time as $\eta_0 [1 + k(\Delta v) \mathcal{A} t]$, where η_0 is the seed amplitude, k is the wave number, Δv is the change in velocity due to the shock, and \mathcal{A} is the Atwood number [24]. Initial roughness of the inner surface is less than or equal to the roughness of the outside of the capsule [25], which was measured using atomic force microscopy and

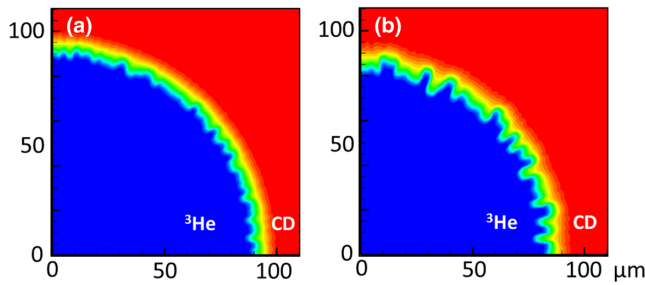


FIG. 2 (color online). Atomic distribution of CD and ^3He from two-dimensional, multimode DRACO simulations (a) when the rebounding shock strikes the fuel-shell interface and (b) near peak compression. Blue indicates pure ^3He ; red, pure CD. Simulations show negligible development of perturbation modes prior to reshock of the fuel-shell interface, and little growth thereafter. The maximum penetration depth of shell material into the gas is approximately $10\ \mu\text{m}$, or approximately 15% of the fall-line depth.

had an rms amplitude less than $0.6\ \mu\text{m}$. Evaluating the expected peak amplitude of RM growth by shock bang time in this experiment, the rms peak amplitude is less than $1\ \mu\text{m}$, which is insufficient to reproduce the observed yields assuming full atomic mix. Additionally, the shock breakout is immediately followed by a period of strong inward acceleration, which will further stabilize RM growth [7].

During the deceleration phase of the 2D-DRACO simulations, instabilities grew to a spike penetration distance of approximately $10\ \mu\text{m}$ [see Fig. 2(b)], which is less than 15% of the minimum shell radius. A useful reference value for the penetration distance is the “fall line,” defined as the projection of the fuel-shell interface location if it were to continue imploding at its peak velocity rather than decelerate. A simulation of the fall-line and fuel-shell interface position versus time is shown in Fig. 3(a). In a hydrodynamic model of the implosion, the fall-line represents the farthest that shell material can penetrate into the gas at any given time [26,27]. A physics-based mix model describing penetration of shell material to a fraction of the fall-line depth was employed, assuming full atomic mixing of shell and fuel in the mix region [28]. This fall-line analysis is unable to recreate the observed yields. Comparing the $< 15\%$ penetration fraction observed in 2D-DRACO simulations to the fall-line analysis as shown in Fig. 3(b), the D^3He -proton yields predicted by these hydrodynamic methods are approximately an order of magnitude lower than observed.

This inability to generate the observed yields even when positing a worst possible case of hydrodynamic mix stems from the fact that in these experiments, hydrodynamic mix is introduced only during the deceleration phase, after peak shock burn. To generate the high values of yield observed, the mix must be established prior to shock burn. This temporal selection makes exploding pushers an excellent

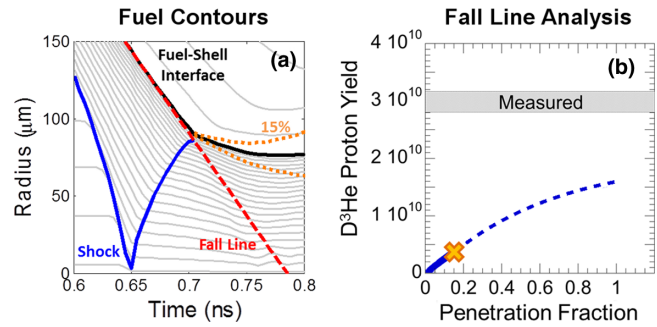


FIG. 3 (color online). (a) Lagrangian mass-element trajectories from a 1D simulation of these experiments. The fall line (red dashed), a tangent projection of the fuel-shell interface (black solid) at peak velocity, represents the furthest distance hydrodynamic shell mix could penetrate into the gas. (b) A fall-line mix model, in which hydrodynamic mix penetrates to a fraction of the fall-line distance from the fuel-shell interface (“penetration fraction”), was unable to reproduce the observed D^3He -proton yields, even in the physically unreasonable worst-case scenario of penetration fraction = 1. Multimode 2D-hydrodynamic simulations of these experiments (Fig. 2) predict a penetration fraction of 15% [(a) orange dotted line, (b) orange x], corresponding to roughly an order of magnitude less yield than was observed.

test bed for physics impacting ICF implosions prior to the deceleration phase.

It has been shown that hydrodynamic growth processes are insufficient to explain the observed yields in these experiments, which implies that another mechanism or mechanisms for mix must be dominant. Ion diffusion provides one driving mechanism for atomic mix at the fuel-shell interface in these experiments that is sufficiently strong to generate the observed yields.

The fuel-shell interface introduces a boundary in partial pressure, such that it is entropically favorable for deuterium, carbon, and ^3He ions to diffuse across it. Classical atomic diffusion will occur at this boundary at a rate governed by the ion species concentration gradient: $j_k = -D\nabla n_k$, where D is the classical diffusion coefficient, $D = (\langle Z \rangle + 1)k_B T / \langle A \rangle \nu_{kl}$ [29], $\langle Z \rangle$ and $\langle A \rangle$ are the local average charge state and ion mass, and ν_{kl} is the collision frequency of ion species k with all species $l \neq k$. The diffusion coefficient scales as $T^{5/2}n^{-1}$. At the fuel-shell interface, shell material is predicted to be cold (30 eV) and dense ($10^{22}\ \text{cm}^{-3}$), implying a small diffusion coefficient ($D \sim 8\ \mu\text{m}^2/\text{ns}$, calculated for shell deuterium in ^3He) prior to shell burnthrough, which occurs at approximately 0.5 ns in these experiments. After shell burnthrough the temperature at the fuel-shell interface increases rapidly to $\sim 1\ \text{keV}$ while the density drops to a few $10^{21}\ \text{cm}^{-3}$, driving much more rapid diffusion: D reaches $\sim 4 \times 10^4\ \mu\text{m}^2/\text{ns}$ in the $\sim 200\ \text{ps}$ before shock burn, with an average value of $\sim 4 \times 10^3\ \mu\text{m}^2/\text{ns}$. Using the solution for Fick’s law in a planar slab as an approximation to the spherical case, the depth of the mix layer grows as $\sqrt{4Dt}$ and the depth of

deuterium mix into the ^3He is expected to be on the order of tens of microns.

Simulations of the experiments performed with the 1D-radiation-hydrodynamics code HYADES [30] were postprocessed using this formalism to determine the impact of classical atomic diffusion on ion density profiles. The ion density flux at each zone boundary was calculated while conserving ion number globally; the pressure and temperature profiles were not modified. Ion flux at each time step was limited to 10% of the ions in the source zone to prevent numerical instability. Figure 4 shows that by shock bang time, a mix layer with a full width at half maximum in excess of $10\ \mu\text{m}$ is expected to develop at the fuel-shell interface. The simulated yield from this mix layer is approximately 70% of the observed values. The total amount of mixed deuterium in this simulation is 15% of the total ^3He , within a factor of a few of the earlier estimate from hydroequivalence.

The effect of ion diffusive mix is predicted to be much less significant for implosions filled with 50:50 D^3He , for which simulated DD-neutron and D^3He -proton yields vary by less than 5% when the ion-diffusion postprocessing is applied. After shell burnthrough the deuterium density is comparable in the 50:50 D^3He fuel and the remaining CD plasma, so the partial pressure gradient driving deuterium diffusion vanishes. This finding supports the assumption that mix is negligible in the D^3He implosion, which was used to estimate the approximate amount of deuterium mix.

In plasmas with multiple ion species, the diffusion rate will also include terms associated with gradients in pressure, temperature, and electric potential [29]. These sources

may further enhance atomic mix as shocks traverse the fuel-shell interface. A 1D-radiation-hydrodynamic simulation of the ^3He -filled experiments incorporating an integrated ion-diffusion model, including pressure and electron-pressure gradient terms, generated D^3He proton yields approximately $2\times$ the observed yields. Using the same code to simulate a $20\text{-}\mu\text{m}$ -thick CD shell under the same laser and gas fill conditions resulted in negligible mix at shock burn and a D^3He -proton shock yield 4 orders of magnitude smaller than the thin CD shell case. Since the thick CD shell does not burn through during the implosion, the fuel-shell interface remains cold and dense: the average diffusion coefficient of shell deuterium into the ^3He is estimated to be $D \sim 5\ \mu\text{m}^2/\text{ns}$ over the 1.6 ns prior to shock burn. This value is 3 orders of magnitude smaller than the value in the thin-shell case, and the estimated depth of the mix layer generated by diffusion is 10 times smaller. Ion diffusion provides an explanation for the high shock yields in these experiments that is consistent with the observed lack of mix at shock yield in thick-shelled experiments [15].

These findings indicate that ion diffusion, a kinetic plasma effect, plays an important role in these experiments, and imply that such effects must be considered in plasma systems containing comparable temperatures, densities, and gradients. In ignition experiments, the fuel-shell interface remains at low temperature and high density throughout the implosion, and ion diffusion is not expected to significantly contribute to fuel-shell mix. However, the initial shock in the central gas of ignition targets generates temperatures and densities comparable to those in exploding pusher implosions. Strong gradients at the shock front may drive ion species separation via diffusion, which alters the initial conditions for fuel compression and burn. Diffusion and other kinetic effects may also affect the transfer of mass from the cold DT fuel into the hot spot during compression. This work provides an experimental constraint on models relevant to these kinetic effects.

In summary, measurements of nuclear yield from direct-drive implosions of thin CD shells show the same yield from capsules filled with pure ^3He and from capsules filled with a hydroequivalent 50:50 mixture of deuterium and ^3He . The observed yields indicate that the number density of deuterium in the ^3He gas is of the order of 10% the ^3He number density before fusion burn. Hydrodynamic mix mechanisms have been ruled out as explanations for this observation, as they do not introduce sufficient fuel-shell mix prior to shock rebound. Integrated 1D-radiation-hydrodynamic calculations including ion diffusion indicate that the amount of mix generated by ion diffusion is sufficiently high to explain the observations. In future experiments, nuclear burn region images using fusion protons and neutrons will be obtained to better constrain modeling of the observed mix process. Fully kinetic simulations of such implosions, in particular during the

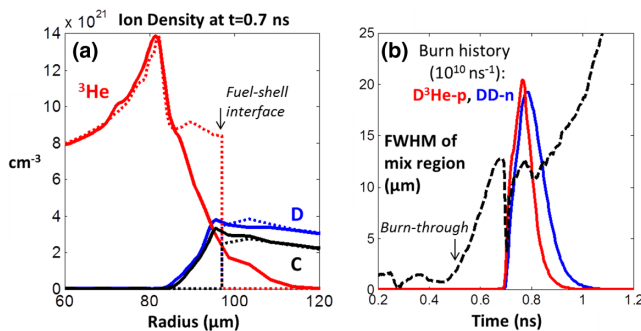


FIG. 4 (color online). (a) Ion density profiles from 1D simulations of a thin CD experiment, just prior to the rebounding shock passing through the fuel-shell interface. Dotted curves show the raw hydrodynamic output with no diffusion; solid curves show the same simulation postprocessed with a classical ion diffusion model. The D^3He -proton yield generated from the ion-diffusive mix layer is within a factor of 2 of the observed yield. (b) Full width at half maximum of the mix region (black dashed) and nuclear burn rate for DD- n (blue) and D^3He - p (red) production as a function of time in the postprocessed ion-diffusion simulation. The thickness of the mix region rapidly increases after shell burnthrough (0.5 ns), exceeding $10\ \mu\text{m}$ just prior to shock burn (0.7 ns).

dynamic epoch of shock breakout across the fuel-shell interface, will be highly informative in terms of understanding the mechanics of mix in these implosions, and their application to other experiments of interest.

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